Algorithms for Random Variate Generation

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November 2023

1 Introduction

Random variate generation refers to the process of generating random numbers or values that follow a specific probability distribution. This is a fundamental concept in statistics and probability theory, and it has various applications in fields such as simulation, modeling, and data analysis.

Here we have some common algorithms for generating random variates from different distributions:

Uniform Distribution

Algorithm:

$$X = \text{lower} + (\text{upper} - \text{lower}) \times \text{random number between 0 and 1}$$

Normal (Gaussian) Distribution

Box-Muller Transform:

$$\begin{aligned} Z_0 &= -2\ln(U_1)\cos(2\pi U_2) \\ Z_1 &= -2\ln(U_1)\sin(2\pi U_2) \\ X &= \text{transform}(Z_0) \end{aligned}$$

where U_1 and U_2 are independent random numbers uniformly distributed in (0,1), and Z_0 and Z_1 are independent standard normal variates. You can then transform Z_0 to get X.

Exponential Distribution

Inverse Transform Method:

$$X = -\frac{1}{\lambda}\ln(U)$$

where U is a random number uniformly distributed in (0,1), and λ is the rate parameter.

Poisson Distribution

Inverse Transform Method:

$$X = \left\lfloor -\frac{1}{\lambda}\ln(U) \right\rfloor$$

where U is a random number uniformly distributed in (0,1), and λ is the Poisson parameter.

Binomial Distribution

Inverse Transform Method: Use the cumulative distribution function (CDF) of the binomial distribution to find the random variate.

Gamma Distribution

Marsaglia and Tsang Method: This is an efficient method for generating random variates from a gamma distribution. It involves a combination of normal random variates and the gamma distribution.

Beta Distribution

Transformation Method: The beta distribution can be generated using the cumulative distribution function (CDF) and its inverse.

Lognormal Distribution

Transformation Method: If X follows a normal distribution, then e^X follows a lognormal distribution.